devices. This corresponds to a minimum noise figure of 0.34 dB. The minimum noise figure at 18 GHz for a 0.1 μm device, projected in an identical manner, is 1.2 dB.

VI. Conclusions

Reported device performance from the technical literature published from 1966 to 1988 was analyzed to predict ultimate frequency limits of GaAs MESFET's. The data indicate that gain–bandwidth products in the range of 200 GHz and maximum frequencies of oscillation of the order of 700 GHz may be achievable with GaAs MESFET structures. Previous work [2] indicates that if progress continues at the present rate, such performance will be achieved by the year 1997. Achieving the projected performance will almost certainly require advances in current process technology. Although 0.1 μm devices have already been fabricated, optimally scaled devices with superior material quality and an absence of surface and channel-interface states have not been achieved. Further advances in frequency performance will be possible with other solid-state transistors such as InP FET's or HEMT's, which were not considered to form the data base for this study.

The data presented can also be used as a standard upon which to judge device scaling efforts. If appropriate device scaling has been achieved, GaAs FET figures of merit should fall on or above the curves presented here.

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A Method of Tolerance Enhancement for Filters and Amplifier Matching Networks

A. N. Riddle and R. J. Trew

***Abstract*—A new filter prototype for increasing the tolerance of passive networks to load variations is presented. A method based on the way in which a network's reflection coefficient changes in response to component and load reactance variations is used to develop the filter polynomial. This new filter polynomial has greater tolerance to load reactance variations, component variations, and finite element Q than Butterworth, Chebyshev, or elliptic structures. Examples using this new filter for tolerance enhancement of filters and matching structures are presented.**

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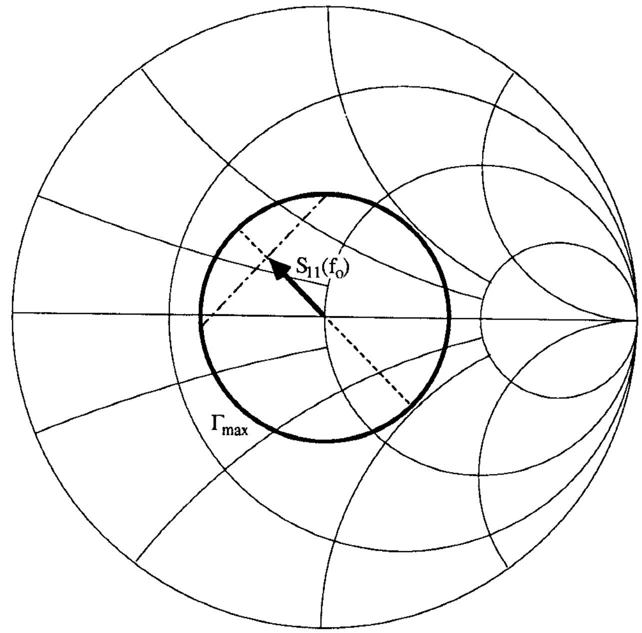

Fig. 1. A Smith chart containing the maximum tolerable reflection, Γ_{max}, and the input reflection coefficient at $f_0, S_{11}(f_0)$. The dashed radial line represents the direction of minimum absolute variation in $S_{11}(f_0)$. The alternating line perpendicular to the $S_{11}(f_0)$ vector represents the direction of maximum absolute tolerance since the distance between $S_{11}(f_0)$ and the $S_{11\,max}$ circle is maximized.

I. Introduction

To date little work has been reported on appropriate design procedures for high-tolerance matching networks. A new method for designing suitable matching networks is presented in this paper. In particular, a solution to the approximation problem for deriving filters with greater tolerances to load parasitics, loss, and element tolerances is presented. The filter polynomial represented in this paper is intended to be used with synthesis procedures discussed elsewhere [1], [2]. Only lumped element prototypes are considered so that the effects of the filtering function, rather than a particular realization method, may be studied.

The first detailed consideration of ideal responses for matching filters was presented by Fano [3]. Although Fano demonstrated that a low-ripple Chebyshev response approximating a constant mismatch was superior to a large-ripple Chebyshev response with the same peak mismatch, he did not explore filter responses other than Butterworth, Chebyshev, and elliptic types.

The filter polynomial developed in this paper was designed to maximize the filter's tolerance to load reactance variations. Load reactance variations typically would be changes in FET input capacity or bond wire inductance. Since the sensitivity to reactance variations is reduced, both lead inductance and device capacitance variations have less effect on the amplifier response when this filter prototype is used to design the matching network.

This new filter response was derived from geometrical considerations in the reflection coefficient plane (Fig. 1). Amplifiers and filters are specified not to exceed a certain reflection coefficient. Let this specification be the Γ_{max} circle shown in Fig. 1. Tolerant networks allow greater variations in circuit components before the response exceeds this Γ_{max} circle. The $S_{11}(f_0)$ vector shown in Fig. 1 represents a circuit's response at one frequency. When component variations force this vector radially outward, we find the minimum tolerance to component variations. Since real components vary about a mean value we should consider the minimum positive or negative change in a component that violates the specification. The geometry of Fig. 1 shows that the maximum tolerance occurs when component changes cause changes in $S_{11}(f_0)$ perpendicular to the radius (i.e., circumferentially). This paper focuses on a heuristically derived filter polynomial which translates reactive changes in the filter's load to approximately circumferential changes in S_{11}.

II. Theory

By analyzing how various filtering functions fall out of specification as the filter load is changed, several conclusions may be reached. Load reactance variations tend to change the center frequency of the Butterworth filter but not the magnitude of the reflection. This would create an ideal tolerance characteristic if it were not for the band edge variations. A Chebyshev filter has the peaks of its ripple affected by load reactance variations, but has more margin for error at the band edges. The new filter polynomial presented here was derived to have the midband tolerance characteristics of a Butterworth filter and the band edge characteristic of a Chebyshev filter.

The filtering polynomial presented in this paper (the F characteristic) has a response similar to a second-order Chebyshev filter. However, the F filter has this single ripple characteristic for all orders (Fig. 3), and so is not a Chebyshev filter. The broad midband response dip has a high tolerance to load reactance variations (as does the Butterworth filter), and the response peaks at the band edges help sharpen the skirts of the filter. The multirippled characteristic of the Chebyshev filter is avoided since as more ripples are added the response becomes more sensitive to load reactance variations. This new characteristic exists only for second- and higher order filters. At the second order it degenerates to a Chebyshev characteristic. The F filter has its highest tolerance when designed to have a flat mismatch and a small amount of ripple. Guidelines for the mismatch and the amount of ripple will be given later.

For a third-order response the F filter polynomial is

$$F_{3\alpha}(p)=\frac{\beta}{\alpha^2}p^3+\frac{1}{\alpha^2}p^2+\beta p+1 \quad (1)$$

where α is the distance to the $S_{11}(f)$ zeros (matched case) in the normalized filter,

$$\beta=\frac{\sqrt{2\alpha^2-1}}{1-\alpha^2} \quad (2)$$

$p=\sigma+j\omega$, and $2^{-1/2}\leqslant\alpha\leqslant 1$. The following properties characterize this filter polynomial:

$$F_{n\alpha}(0)=1.0$$

$$F_{n\alpha}(j\alpha)=0.0$$

$$F_{n\alpha}(j)=1.0 \qquad \text{for the matched case.}$$

The polynomial in (1) is derived by multiplying out

$$\frac{\beta}{\alpha^2}(p^2+\alpha^2)\left(p+\frac{1}{\beta}\right)$$

and requiring that the above conditions be satisfied. The above polynomial should be used as the characteristic function of a filter, just as a Chebyshev polynomial would be used. The fourth-order equation is derived similarly to the third-order polynomial. These ideas lead to the following fourth-order filter polynomial:

$$F_{4\alpha}(p)=\frac{1}{\alpha^2}p^4+\frac{1}{1-\alpha^2}p^3+\frac{\alpha^2+1}{\alpha^2}p^2+\frac{\alpha^2}{1-\alpha^2}p+1. \quad (3)$$

TABLE I

$F_{n\,78}$ Filter Values for $\Gamma_{max}=0.23$ and $\Gamma_{min}=0.115$

Order	R_S	L_1	C_2	L_3	C_4	R_L
2	1.0	1.23968	0.52601			1.5974
3	1.0	1.43152	0.83208	0.85453		1.5974
4	1.0	1.474	0.92859	1.6119	0.24289	1.5974

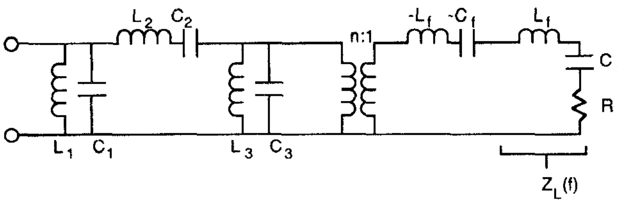

Fig. 2. The third-order lumped filter network used in examining the tolerances of filter responses. Inductors and capacitors 1 through 3 represent lossless filter elements. R, C_f, and L_f represent a narrow-band model of an FET input impedance. The ideal transformer and negative elements are used to equalize the filter for the complex FET impedance without disturbing the filter components.

Higher orders may be obtained using the following recursion formula:

$$F_{(n+1)\alpha}=(p^2+1)F_{(n-1)\alpha}+pF_{n\alpha} \quad (4)$$

where n is the order of the filter. The recursion formula is derived by requiring that the above properties be met. The parameter α is chosen to maximize the filter tolerance, which could be done through least mean square optimization techniques. A value of 0.78 serves as a good starting point for the optimization. Normalized filter element values are given in Table I for $n=2$, 3, and 4 with $\alpha=0.78$.

III. Results

The network of Fig. 2 is used to examine the tolerances of the Butterworth, the Chebyshev, and the F filter to variations in a series load capacitance. In order to give a fair comparison between the Chebyshev and F filter types, each filter is designed to have a bandwidth such that

$$\int_0^\infty \ln\left|\frac{1}{S_{11}(\omega)}\right| d\omega=K \quad (5)$$

where K is the same for all filters. Both filters, therefore, have the same ability to absorb complex load reactances. Since Butterworth filters are known to be suboptimal [3], no attempt is made to compare the Butterworth filters on the basis of matching ability.

In the network shown in Fig. 2 the load is a narrow-band model of the input impedance of a 0.7×250 μm FET. The model is intended for the 3.7 to 4.2 GHz range. L_f, C_f, and R are 1.4 nH, 0.3 pF, and 8 Ω, respectively. Note that ideal responses were obtained by neutralizing the load reactances with negative components and using an ideal transformer. This is so that component tolerances could be established without working through the details of a specific realization. In practice, Youla's broad-band equalization theory [7] would be used to neutralize the load reactances, and the ideal transformer could be absorbed into the filter by Norton's transformation and the use of immittance inverters.

In order to test the concepts and formulas developed in the theory section, various filters will be examined. These band-pass

TABLE II
FILTER ELEMENT VALUES ($f_0 = 3.942$ GHz)

Type	BW (GHz)	n	C_1 (pF)	L_1 (nH)	G_1 (mS)	L_2 (nH)	C_2 (pF)	R_2 (Ω)	C_3 (pF)	L_3 (nH)	G_3 (mS)
Chebyshev	0.5	2.5	8.729	0.1867	8.648	18.11	0.09	17.94	8.729	0.1867	8.64
Chebyshev (mismatched)	0.562	2.22	8.585	0.1899	8.51	13.99	0.1165	13.86	7.005	0.2327	6.94
New Filter ($F_{3\,78}$ mismatched)	0.53	1.98	8.594	0.1897	8.52	12.49	0.131	12.37	5.138	0.3173	5.09

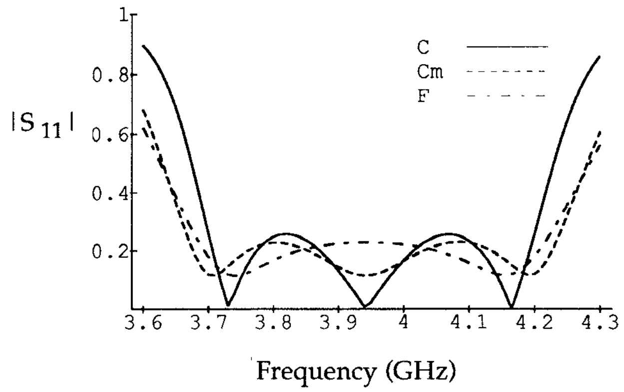

Fig. 3. S_{11} versus frequency for the three filters described in Table II. The solid line is the Chebyshev. The dashed line is the mismatched Chebyshev. The alternating line is the $F_{3\,78}$ filter. All of the filters have the same return loss–bandwidth product.

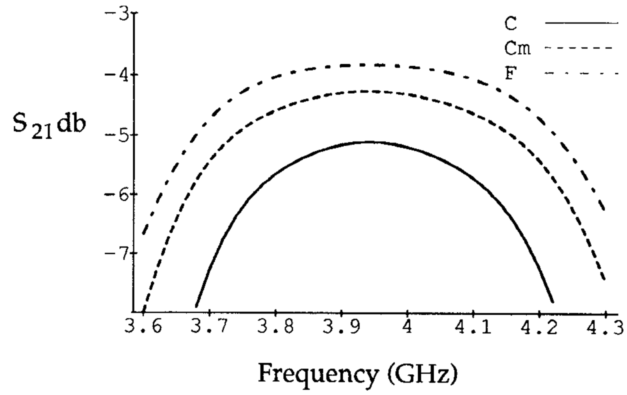

Fig. 4. Gain versus frequency plot for the filters described in Table II having lossy elements with Q's of 25. The Chebyshev response has a solid line, the mismatched Chebyshev has a dashed line, and the $F_{3\,78}$ response has an alternating line.

filters are shown in Fig. 3. These networks are analyzed over the 3.7 to 4.2 GHz range for tolerances to load variations, element variations, and loss. Table II gives the element values for a Chebyshev filter of 0.5 GHz bandwidth and $\rho_{max} = 0.26$, a mismatched Chebyshev filter of 0.5617 GHz bandwidth with $\rho_{max} = 0.23$ and $\rho_{min} = 0.115$, and a mismatched F filter with 0.53 GHz bandwidth, $\alpha = 0.78$, $\rho_{max} = 0.23$, and $\rho_{min} = 0.115$. This α value is optimized for highest tolerance. All of the filters have a center frequency of 3.942 GHz. All of the above filters (except the Butterworth) also have a K of approximately 7.4×10^9. For lossy filters the conductances and resistances of Table II appear in parallel or in series with their respective resonant circuits shown in Fig. 2.

A tolerance analysis of these filters showed that the mismatched Chebyshev filter and the F filter have up to 15 and 50% increases in load tolerance over the Chebyshev filter, respectively. The mismatched Chebyshev filter and the F filter exhibit increases in the element tolerances up to 3 and 4.5 times greater than that of a Chebyshev filter. This shows the F filter has a significant increase in element and device tolerances with respect to commonly used filter prototypes. The tolerance information is given for a maximum $S_{11}(f) = 0.333 = \Gamma_{max}$ with only one component being varied. The improvement of the mismatched Chebyshev response over the normal Chebyshev response may be attributed to the reduction of its maximum reflection coefficient. The improvement of the F filter over the mismatched Chebyshev is because the F filter has fewer ripples. The increases in tolerance to parameter variations should result in circuits which are easier to tune.

It is important for noise figure and circuit performance that the filters be affected as little as possible by loss. Fig.4 demonstrates the effects of uniform element Q's of 25 on the three filter types discussed. In both reflection and transmission responses, the F filter is least affected by loss. The F filter gives 0.4 and 1.3 dB less loss at the band center than the mismatched and normal Chebyshev filters, respectively. Note that predistortion would help the band edge response but hurt the band center response. The end result is that the F filter is less sensitive to element Q than either of the Chebyshev filters.

IV. CONCLUSIONS

A new filter prototype, the F filter, has been developed. This filter has greater tolerance to load reactance and component variations than Chebyshev filters. This new filter prototype was based on geometric considerations in the reflection coefficient plane. While the F filter prototype was designed specifically for a greater tolerance to load reactance variations, the F filter also showed greater tolerance to the effects of loss. This makes this new filter prototype useful for several reasons. Since insensitivity to loss and the greatest possible tolerances are essential to designing lower cost and easily tuned microwave circuits, the design techniques described above should aid the production of microwave circuits.

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